Newman Calculations Guide

"Through our Maths curriculum, we aim to develop resilient and independent learners who enjoy Maths, are fluent in the fundamentals, can reason mathematically, are adept at problem solving and well-prepared for their next steps in life."

All staff at Newman Catholic College are committed to improving Numeracy.

Credit: Wade Deacon High School for their work on this calculations guide.

Table of Contents

Introduction 4
Aims of this booklet: 4
How to Use this Booklet: 4
Numeracy across the Curriculum Policy 2014 6
Common Methodology
NUMBER
Place Value 10
Square numbers 11
Estimation and rounding 12
Rounding Whole Numbers 12
Rounding to Decimal Places 12
Rounding to Significant Figures 13
Operations and Calculations
Addition 14
Subtraction 15
Multiplication of Whole Numbers 15
Multiplication 16
Multiplying larger numbers 17
Division 19
Order of Calculation (BIDMAS) 19
Negative Numbers: 20
FRACTIONS 22
Equivalent Fractions 22
Simplifying Fractions 23
Calculating Fractions of a Quantity 24
Percentages 25
Finding Percentages 26
Expressing something as a percentage 27
Ratio 27
Simplifying Ratios 28
Sharing in a given ratio 30

Money & Decimal Places 30
PROBLEM SOLVING 33
Shape, Space and Measures 36
Time 36
Interpreting Timetables 38
Measurement 40
Reading scales 40
Converting between units 41
Metric and imperial units 42
Perimeter 43
Area 44
Volume 45
Statistics
10 Commandments for drawing or plotting a graph 46
Data Tables 47
Bar Graphs 47
Line Graphs 48
Scatter Graphs 49
Pie Charts 50
Drawing Pie Charts 52
Averages 52
Probabilities 54
Glossary of Terms 56

Introduction

Aims of this booklet:

- To enable all teachers and parents to adopt a common approach to numeracy methods across all curriculum areas.

- To enable pupils to more easily recognize the numeracy skills required for their work and will ensure consistency in the methods they will use.

- To enable Parents to build confidence in currently taught methods in order to support their child in Numeracy across all curriculum areas.

How to Use this Booklet:

- The Numeracy Policy highlights the importance of ensuring Numeracy skills are taught through all curriculum areas, together with indicating how teachers should ensure areas of Numeracy are identified in their planning and highlighted to pupils.

- The Numeracy Methods section breaks down the key skills into the 4 core areas of Numeracy.(defined in the School Numeracy Policy). Examples are used to show the current widely used methods. In most cases there is more than one method. Teaching staff and Parents should be guided by the method which the child feels most confident with, rather than insisting on one particular strategy.

- This guide is not supposed to be exhaustive, but as a guide to current methodology. All topics are not covered, if any further information or support is required, please contact your subject Numeracy Coordinator or a member of the Maths team.

Numeracy across the Curriculum Policy 2022

"Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen". (PISA)

"Numeracy is a life skill – necessary to allow each of us to make informed choices and decisions in all aspects of everyday life. This can be at home, at work, as a consumer, as a parent. It touches activities not only such as choosing a mortgage or a utilities contract, checking invoices or wage slips, working out a monthly budget or helping with children's homework, but also planning a journey, cooking a meal, reading a newspaper, or playing sport. Confidence and competence in numeracy affect all aspects of our lives, every day." (The Mathematical Journey-National Numeracy)

Ofsted

Framework for School Inspection – Descriptors for an Outstanding School:

- *There is excellent practice that ensures that all pupils have high levels of literacy and mathematical knowledge, understanding and skills appropriate to their age.*
- *The teaching of reading, writing, communication and mathematics is highly effective and cohesively planned and implemented across the school curriculum*
- *The school's actions have secured improvement in achievement for disadvantaged pupils, which is rising rapidly, including in English and mathematics*

- *Pupils make substantial and sustained progress throughout year groups across many subjects, including English and mathematics, and learn exceptionally well*.
- *Pupils acquire knowledge and develop and apply a wide range of skills to great effect in reading, writing, communication and mathematics. They are exceptionally well prepared for the next stage in their education, training or employment.*

Aims and Rationale

"Across the UK, **around 4 in 5 adults** have a low level of numeracy - roughly defined as the adult skills equivalent of being below GCSE grade C level. In 2011, the Skills for Life Survey showed that **numeracy skills in England declined** in the 8 years from 2003, whereas literacy improved. The difference between the two areas was already large - on average people tend to be at least one level better at literacy than in numeracy. These findings led to the realisation that 17 million adults in England are working at a level roughly equivalent to that expected of children at primary school. Around **30% of the people who rated their skills as "very good" performed poorly** - showing a sizable lack of awareness of this problem." (a. St Clair, Ralph, Lyn Tett, and Kathy Maclachlan. 2010).
"The maths they are taught at school does not necessarily overlap with the maths that can best help them later in life. **Among 16 to 24 year olds who passed their GCSEs with a C or above, only 24% were at the adult equivalent – Level 2** " (Department for Business Innovation and Skills. 2012. "Skills for Life Survey 2011).

The numeracy programme will aim to prepare students for everyday life through regular practical application of mathematics across a diverse range of activities and contexts delivered by all curriculum areas.

The policy will ensure that:

- All classroom teachers and support staff have responsibility for promoting numeracy within their specialist subject.

- Leaders utilise the Better Together Programme to monitor and review the learning and teaching of Numeracy across the Curriculum within each school of excellence.
- Staff are given appropriate training to ensure that Numeracy opportunities are addressed within lessons.
- Where pertinent, Classroom teachers support the whole school's management of numeracy through highlighting numeracy in schemes of work and delivering numeracy content using consistent formulas, terms and methods.
- School Leaders and numeracy ambassadors rigorously monitor and review how effectively teachers are developing pupils' Numeracy skills and identify priorities for Numeracy within their subject areas.
- Numeracy Ambassadors promote the numeracy within their subject areas and ensure they are a key feature of day to day Learning and Teaching.

Essentials of Numeracy: Day to Day Learning and Teaching

The Teaching of Numeracy should not be considered an 'add on' in lesson planning and any Numeracy opportunity that arises should be addressed.

In addition to Mathematics lessons, students should be supported across the curriculum in the four essentials of numeracy:

Numbers, Operations and Calculations, Handling Information, Shape, Space and Measures

Numbers

This includes:

Whole numbers , size and order (comparing, ordering), sequences and patterns (odd/even, square, prime etc), place value(money context, measures, estimation), Numbers "in between" whole numbers (fractions, percentages, decimals), using numbers (for measuring, counting, ration, proportion)

Operations and Calculations

This includes:

Addition and Subtraction, Multiplication and Division, effective use of Calculators,

Handling Information

This includes:

Graphs and Charts, Probability, Processing Data, Types of Data, Comparing Sets of Data

Shape, Space and Measures

This includes:

Shape and Space (symmetry, making and drawing, 2d/3d shapes, reflection, translation, rotation), Measurement (units of, area, volume, perimeter)

To achieve this, classroom teachers need to be aware of which elements of Essential Numeracy they currently deliver or can plan to deliver in the future.

Teachers should have a knowledge of how these Essentials are delivered to pupils in mathematics in order to achieve whole school consistency, and of how the Essentials contribute to success in their own subject area.

This includes having the confidence to give pupils access to appropriate methods and terminology to improve their numeracy and their subject specific skills and understanding.

All teaching and support staff should promote positivity around Numeracy. Pupils should be encouraged to develop a love of number and recognise the importance of Numeracy in the world around them.

COMMON METHODOLOGY

Place Value

- Every number can be 'partitioned' into its component parts

e.g. 2,465.12 = 2000 + 400 + 60 + 5 + 0.1 + 0.02

The Units column is the single digits, followed to the left by tens, hundreds, thousands,ten thousands, hundred thousands, millions etc.

0.1 = 1 tenth, tenths are the first column after the decimal point. There are ten tenths in a whole.
0.01 = 1 hundredth. There are ten hundredths in a tenth.

When dealing with numbers, always ensure the columns are lined up on top of each other including the decimal point which should be on top of each other.

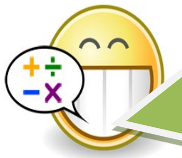
e.g.		123.49	NOT	123.49
	+	36.4		36.49

Square numbers

Square numbers are the result of multiplying a number by itself.
e.g. 1x1 = 1, 2x2=2
These are written using powers e.g. $4 \times 4 = 4^2$

They can be used in many areas of Maths including finding Area of circles.

Estimation and rounding



We can use rounded numbers to give us an approximation. We can then use this to estimate the answer to a calculation. This allows us to check that our answer is sensible. We generally round using the first non-zero digit i.e. 1st significant figure.

Rounding Whole Numbers

Numbers can be rounded to give an approximation, either up or down. In general, to round a number, we must first identify the place value to which we want to round. We must then look at the next digit to the right (the "check digit") - if it is 5 or more round up.

Example Round 46 753 to the nearest thousand.

6 is the digit in the thousands column - the check digit (in the hundreds column) is a 7, so round up.

46 753 = 47 000 to the nearest thousand

Rounding toDecimal Places

Example 1 Round 1.57359 to 2 decimal places

The second number after the decimal point is a 7 - the check digit (the third number after the decimal point) is a 3, so round down.

1.57359

= 1.57 to 2 decimal places

Rounding to Significant Figures

Numbers can also be rounded to a given number of significant figures. Start with the first non-zero number. This is the 1st significant figure.

Example 2 Round 0.15273 to 2 significant figures

The first significant figure is 1 in the tenths place
The second significant figure is 5 in the hundredths place

0 . 1 5 2 7 3

We then look at the next number and decide whether to round the 5 up or keep it the same. It is 2 so we keep the 5 the same

= <u>0 . 1 5 to 2 significant figures</u>

Operations and Calculations

Addition and Subtraction, Multiplication and Division,

Addition

Mental strategies – There are a number of strategies to complete mentally

Example Calculate 54 + 27

Method 1 Add tens, then add units, then add together

50 + 20 = 70 4 + 7 = 11 70 + 11 = 81

Method 2 Split up number to be added into tens and units and add separately.

54 + 20 = 74 then 74 + 7 = 81

This can also be written on a number line, adding 20 to 54, then 7 to 74.

Written Method

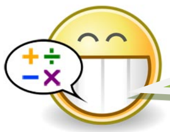
When adding numbers, ensure that the numbers are **lined up** according to place value. Start at right hand side, write down units, carry any tens as 1.

Example I spend £3032 a year on my car loan. My insurance is £589. How much is this in total?

METHOD 1

3032		3032		3032		3032
+589	→	+589	→	+589	→	+589
1		21		621		3621
1		1 1		1 1		1 1
2 + 9 =		3+8+1=1		0+5+1=		3 + 0 =

Subtraction



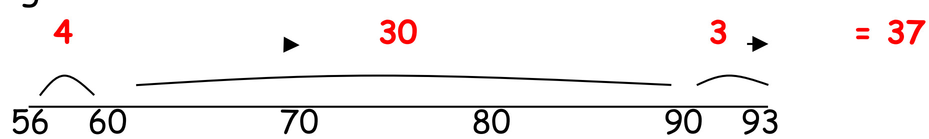
Traditional Column Method can be used, but 'Number Line' Method can be linked to Addition, to also complete Subtraction. This links with Partitioning too!

Mental/Written Strategies

Example Calculate 93 - 56

Method 1 Counting on a Number line

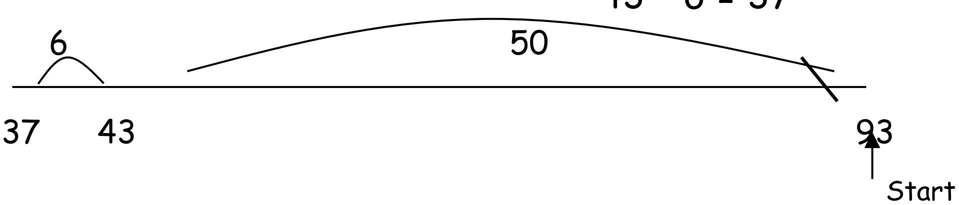
Count on from 56 until you reach 93. This can be done in several ways e.g.



Method 2 Break up the number being subtracted

e.g. subtract 50, then subtract 6   93 - 50 = 43

43 - 6 = 37



Column Method

Example 1 4590 - 386

```
 4590
- 386
 4204
  8 1
```

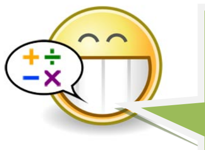
Example 2 Subtract 692 from 14597

```
 14597
-  692
 13905
   3 1
```

Multiplication of Whole Numbers

The Times tables up to 12's should be known. These can be used to find any other multiplication sum.

Mental Strategies



You should use the times tables up to 12 x, to help you answer harder questions

Example Find 39 x 6

Method 1

30 x 6
= 180

9 x 6
= 54

180 +
54

Method 2

40 x 6
=240

40 is 1 too
many so
take away

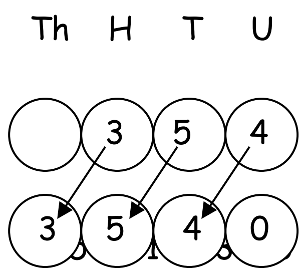
240 - 6
= 234

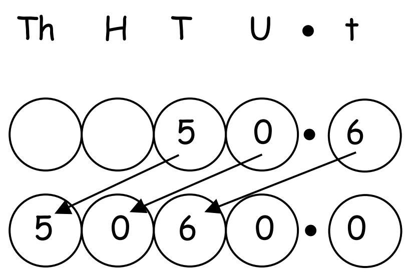
Multiplication

Multiplying by multiples of 10 and 100

To multiply by **10** you move every digit ***one*** place to the left.
To multiply by **100** you move every digit ***two*** places to the left.

Example 1 (a) Multiply 354 by 10 (b) Multiply 50.6 by 100





50.6 x 100 = 5060

(c) 35 x 30

To multiply by 30, multiply by 3, then by 10.

35 x 3 = 105
105 x 10 = 1050

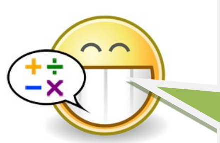
so 35 x 30 **= 1050**

(d) 436 x 600

To multiply by 600, multiply by 6, then by 100.

436 x 6 = 2616
2616 x 100 = 261600

so 436 x 600 **= 261600**



We can also use these rules for multiplying decimal numbers.

Example 2 (a) 2.36 x 20

2.36 x 2 = 4.72
4.72 x 10 = 47.2

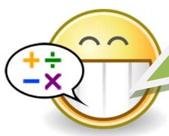
so 2.36 x 20 = 47.2

(b) 38.4 x 50

38.4 x 5 = 192.0
192.0x 10 = 1920

so 38.4 x 50 = 1920

Multiplying larger numbers



There are a number of methods including mental methods like those above. The most commonly taught method is now the grid method. If a pupil is confident at column multiplication, and is always accurate, they should continue to use this method. If mistakes occur, they should try grid method.

Example

There are 35 seats in a row, and 37 rows of seats. Work out if there are enough seats for 1100 people, or will more rows need to be added?

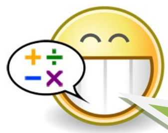
Method 1

Grid Multiplication – This is now the most consistently used method at Secondary level. It uses the smaller multiples to build up larger multiplication sums.

X	30	7	
30	=30 x 30 =900	= 7 x 30 =210	= 900 +210 = 1110
5	=30 x 5 =150	=7 x 5 = 35	= 150+35 = 185 1110 + 185 = 1295

- ✓ **Partition the numbers into tens and units.**
- ✓ **Multiply the values 'on the edges',**
- ✓ **Add up the boxes.**

Division



Division is the opposite of multiplication. You should be able to divide by a multiple of 10 or 100 by moving the numbers opposite to that a single digit without a calculator.

Written Method

Example 1 There are 192 pupils in first year, shared equally between 8 classes. How many pupils are in each class?

$$\begin{array}{r} 2\ 4 \\ 8\ \overline{)\ 1\ 9^{3}2} \end{array}$$

There are 24 pupils in each class

Example 2 Divide 4.74 by 3

$$\begin{array}{r} 1\ .\ 5\ 8 \\ 3\ \overline{)\ 4\ .\ {}^{1}7\,{}^{2}4} \end{array}$$

When dividing a decimal number by a whole number, the decimal points must stay in line.

Example 3 A jug contains 2.2 litres of juice. If it is poured evenly into 8 glasses, how much juice is in each glass?

$$\begin{array}{r} 0\ .\ 2\ 7\ 5 \\ 8\ \overline{)\ 2\ .\ {}^{2}2\,{}^{6}0\,{}^{4}0} \end{array}$$

Each glass contains
<u>**0.275 litres**</u>

If you have a remainder at the end of a calculation, add a zero onto the end of the decimal and continue with the calculation.

Order of Calculation (BIDMAS)

What is the answer to 2 + 5 x 8 ?

Is it   7 x 8 = 56   or   2 + 40 = 42 ?

The correct answer is **42**.

Calculations which have more than one operation(i.e. x, + - or ÷) need to be done in a **standard** order. The order can be remembered **BIDMAS.**

The rule means Brackets should be done first.

(B)rackets
(I)ndices
(D)ivide
(M)ultiply
(A)dd
(S)ubract

Scientific calculators use this rule automatically, some basic calculators may not, so take care in their use.

Example 1   15 – 12 ÷ 6   BIDMAS tells us to divide first

= 15 - 2

= 13

Example 2   (9 + 5) x 6   BIDMAS tells us to work out the brackets first

= 14 x 6

= 84

Example 3   18 + 6 ÷ (5-2)   **Brackets first**

= 18 + 6 ÷ 3   Then divide

= 18 + 2   Now add

= 20

Negative Numbers:

Adding a negative number is the same as subtracting

Subtracting a negative number is the same as adding.

Using a number line:

To ADD count to the right.

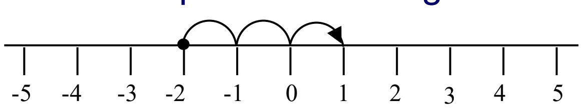
To SUBTRACT count to the left.

Examples:

1. -2 +3

Start at -2

Move 3 places to the right



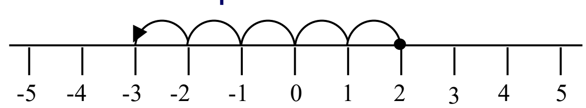
-2 +3 =1

2. 2+(-5)

=2 -5

Start at 2

Move 5 places to the left

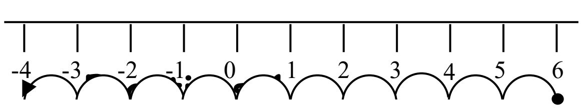


2 +(-5) = 3

3. 6 - 10

Start at 6

Move 10 places to the left



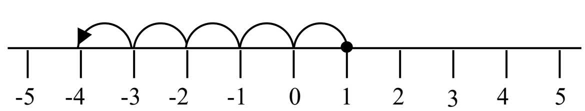
6 -10 = -4

4. -4 -(-5)

= -4 +5

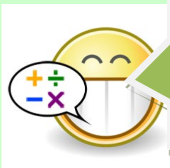
Start at -4

Move 5 places to the right



-4 -(-5) = 1

FRACTIONS

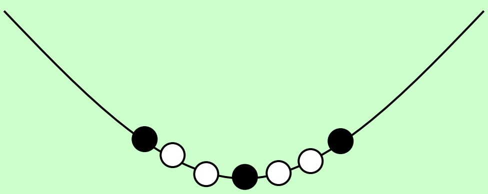


Fractions are used to give a proportion of another value or to state how much of a total something is. For example $\frac{1}{4}$ of my salary goes on my mortgage.

Understanding Fractions

Example

A necklace is made from black and white beads.

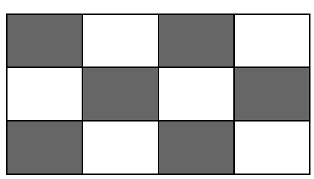


What fraction of the beads are black?

There are 3 black beads out of a total of 7, so $\frac{3}{7}$ of the beads are black.

Equivalent Fractions

What fraction of the flag is shaded?



6 out of 12 squares are shaded.

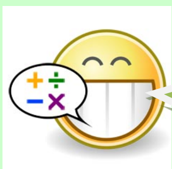
So $\frac{6}{12}$ of the flag is shaded. (6 twelfths)

It could also be said that $\frac{1}{2}$ the flag is shaded.

$\frac{6}{12}$ and $\frac{1}{}$ are **equivalent fractions**.

Simplifying Fractions

Equivalent fractions can be simplified as shown below:



The top of a fraction is called the **numerator**, the bottom is called the **denominator**. Both must be whole numbers. To simplify a fraction, divide the **numerator** and **denominator** of the fraction by the **<u>same number.</u>**

Example 1

(a)

÷5

$$\frac{20}{25} = \frac{4}{5}$$

÷5

(b)

÷8

$$\frac{16}{24} = \frac{2}{3}$$

÷8

This can be done again and again until the numerator and denominator are the smallest possible numbers - the fraction is then said to be in it's <u>simplest form.</u>

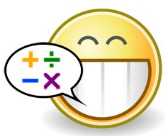
Think of a pizza, 2/3 or pizza is the same as 4/6 of a pizza, only that the slices are bigger or smaller!

Example 2

Simplify $\frac{72}{84}$

$$\frac{72}{84} = \frac{36}{42} = \frac{18}{21} = \frac{6}{7} \text{ (simplest form)}$$

Calculating Fractions of a Quantity



Fractions share amounts into equal parts.

So to find $\frac{1}{2}$ divide by 2, to find $\frac{1}{3}$ divide by 3, to find $\frac{1}{7}$ divide by 7 etc.

Example 1 Find $\frac{1}{5}$ of £150

$\frac{1}{5}$ of £150 = £150 ÷ 5 = **£30**

To find a unit fraction (e.g. $\frac{1}{4}$) divide by the bottom number.

Example 2 Find $\frac{3}{4}$ of 48

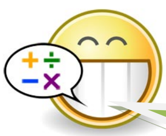
$\frac{1}{4}$ of 48 = 48 ÷ 4 = 12

To find any other fraction, divide by the bottom and then multiply by the top

so $\frac{3}{4}$ of 48 = 3 x 12 = **36**

Percentages

Percentage means 'out of 100'. We divide or multiply to make any value out of 100 to write as a percent. They are widely used to give a way of comparing one value out of another. They can be used by shops(sales & discounts), banks (interest rates), the government(tax rates)



The key percentage building blocks can be used to 'build up' any percentage. They are 100% (all of the amount), 50%, 25%, 10%, 5% and 1%. It is vital to know these to get any harder percentage.

Building Blocks

To get any of the building blocks, divide the amount by the following:

100% - All of the amount you start with

50% - divide by 2

25% - divide by 4 or find 50% and divide by 2

10% - divide by 10

1% - divide by 100.

Some people find using the fraction equivalent easier if they understand, e.g.

25% of £640 = $\frac{1}{4}$ of £640 = £640 ÷ 4 = **£160**

Finding Percentages

Real Life Link: Percentages are used in a variety of places in real life such as Sales in shops, tax on wages, interest on loans, mortgages and bank accounts

Non- Calculator Methods

Example *An Xbox game decreases by 30% from £45. How much will I save?*

Step 1) 'Build the percentage' - 30%= 10% + 10% + 10%

Step 2) Find the percentages. 10% of £45 = 45 ÷ 10

= £4.50

(As there are 10 lots of 10% in 100%).

Step 3) Add the amounts together. £4.50+£4.50+£4.50 = **£13.50**

So **30% of £45 = £13.50**

Example 2 *A £1,200 holiday to Disneyland has a 6% saving for 1 week only, how much will I save?*

Step 1) 'Build the percentage' - 6%= 5% + 1%

Step 2) Find the percentages. 10% of £1,200 = 1200 ÷ 10

= £120

5% of £1,200= 120 ÷ 2 (Half of 10%)

= £60

1% of £1,200 = 1200 ÷ 100 = £12

(as there are 100 lots of 1% in 100%)

Step 3) Add the amounts together. £60 + £12 = £72

So **6% of £1200 = £72**

Expressing something as a percentage

To find a number as a percentage of another number, first make a fraction, this can then be expressed as a percentage by finding that fraction of 100%.

Example 1 There are 30 pupils in Class 3M. 18 are girls.
What percentage of Class 3A3 are girls?

$$\frac{18}{30} = 18 \div 30 = 0.6 = 60\%$$

OR $\frac{18}{30} \times 100\% = \frac{18 \times 100\%}{30} = \frac{1800}{30} = 60\%$

<u>60% of 3A3 are girls</u>

Example 2 James scored 36 out of 44 his biology test. What is his percentage mark?

Score = $\frac{36}{44} = 36 \div 44 = 0.81818... = 81.818..\% =$ **82% (rounded)**

OR $\frac{36}{44} \times 100\% = \frac{36 \times 100\%}{44} = \frac{3600}{44} = 81.818..\% =$ **82% (rounded)**

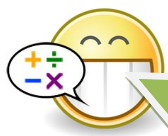
Example 3 In class 2K, 14 pupils had brown hair, 6 pupils had blonde hair, 3 had black hair and 2 had red hair. What percentage of the pupils were blonde?

Total number of pupils = 14 + 6 + 3 + 2 = 25

6 out of 25 were blonde, so,

$\frac{6}{25} = 6 \div 25 = 0.24 =$ **24% were blonde**

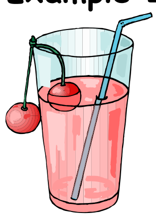
Ratio



When quantities are to be mixed together, the ratio, or proportion of each quantity is often given. The ratio can be used to calculate the amount of each quantity, or to share a total into parts.

Writing Ratios

Example 1



To make a fruit drink, 4 parts water is mixed with 1 part of cordial.
The ratio of water to cordial is 4:1
(said "4 to 1")
The ratio of cordial to water is 1:4.

Order is important when writing ratios.

Example 2

In a bag of balloons, there are 5 red, 7 blue and 8 green balloons.
The ratio of red : blue : green is 5 : 7 : 8

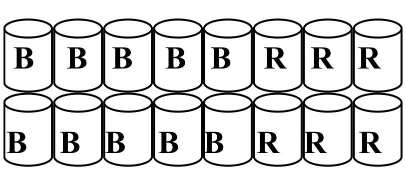
Simplifying Ratios

Ratios can be simplified in much the same way as fractions.

Example 1

Purple paint can be made by mixing 10 tins of blue paint with 6 tins of red. The ratio of blue to red can be written as 10 : 6

It can also be written as 5 : 3, as it is possible to split up the tins into 2 groups, each containing 5 tins of blue and 3 tins of red.



Blue:Red =10:6
=5:3

To simplify a ratio, divide each figure in the ratio by the highest number that goes into both numbers.

Simplifying Ratios (continued)

Example 2

Simplify each ratio:

(a) 4:6 (b) 24:36 (c) 6:3:12

(a) 4:6

= **2:3**

Divide each figure by 2

(b) 24:36

= **2:3**

Divide each figure by 12

(c) 6:3:12

= **2:1:4**

Divide each figure by 3

Example 3

Concrete is made by mixing 20 kg of sand with 4 kg cement. Write

the ratio of sand : cement in its simplest form

Sand : Cement = 20 : 4

= 5 : 1

Using ratios

The ratio of fruit to nuts in a chocolate bar is 3 : 2. If a bar contains 15g of fruit, what weight of nuts will it contain?

Fruit	Nuts
3	2
×5	×5
15	**10**

So the chocolate bar will contain 10g of nuts.

Sharing in a given ratio

Lauren and Sean earn money by washing cars. By the end of the day they have made £90. As Lauren did more of the work, they decide to share the profits in the ratio 3:2. How much money did each receive?

Step 1 **Add up the numbers to find the total number of parts**

3 + 2 = 5

Step 2 **Divide the total by this number to find the value of each part**

90 ÷ 5 = £18

Step 3 **Multiply each figure by the value of each part**

3 x £18 = £54
2 x £18 = £36

Step 4 **Check that the total is correct**

£54 + £36 = £90 √

Lauren received £54 and Sean received £36

Money & Decimal Places

All calculations of money need to be written down to 2 decimal places (two numbers after the decimal point) This could mean that we need to round numbers:

Example 1 Round £1.525 to 2 decimal places

The second number after the decimal point is a 2 - the check digit (the third number after the decimal point) is a 5, so round up.

1.525

= 1.53 to 2 decimal places

We may also need to put in zeros to show our answers to 2 decimal places:

Example 2 Calculate the total cost of the following items

Pencil	20p
Pen	40p
Rubber	30p
Ruler	75p
Sharpener	25p

Total cost = 190p

= £1.90 to 2 decimal places

% Extra Free

Example 1:

A cereal packet usually contains 750g of cereal. There is a special offer packet which contains 25% extra free. How much cereal is in the special offer packet?

Calculate the number of extra grams:

25% of 750g = $\frac{1}{4}$ of 750g
$\frac{1}{2}$ of 750g = 375g
$\frac{1}{4}$ of 750g = 187.5g

Add this to the original number of grams in the packet:

750g + 187.5g = 937.5g

PROBLEM SOLVING

Buy One Get One Free

This offer is usually used when retailers want to clear a large number of items quickly. They are effectively reducing the price of goods by half whilst ensuring that you buy two items at a time.

This offer is only a saving if you would normally use the two items before the goods would be out of date.
If you usually buy one chocolate cake and you get one free, you haven't made a saving you, just have an extra cake. However, if you usually buy two cakes you have made a saving.

Three for the Price of Two

This is similar to the above offer. The retailers are effectively reducing the cost to two thirds of the original price. This offer is only a saving if you would normally use the three items before the goods would be out of date.

Which offer is the best value?

To work this out we need to work out the 'price per one' of something. This can be 100g, 1kg, 1 unit etc. The quantity or amount of each product needs to be the same for a comparison.

Look at the following special offers.

'Swarbricks'	**Brown's Bread**	**Wheaty Bake**
600g	**800g**	**790g**
78p per loaf	**£1.20**	**98p**
3 for 2	**20% extra free**	**10% discount**

a) Which offers the best value for money per gram of bread without the special offer?

Swarbricks: 78p ÷ 600g
= 0.13p per gram

Brown's Bread 120p ÷ 800g
= 0.15p per gram

Wheaty Bake 98p ÷ 790g
= 0.12p per gram

<u>Wheaty Bake is the best value for money at 0.12p per gram</u>

b) Which offers the best value for money per gram of bread with the special offer?

Swarbricks: 3 for the price of 2

Cost = 2 x 78
= 156p

Grams = 3 x 600g
= 1800g

Cost per gram = 156p ÷ 1800g
= <u>0.09p per gram</u>

Brown's Bread 20% extra free

Cost = 120p

Grams = 800g + (20% of 800g)

= 800g + 160g

= 960g

Cost per gram = 120p ÷ 960g

= <u>0.13p per gram</u>

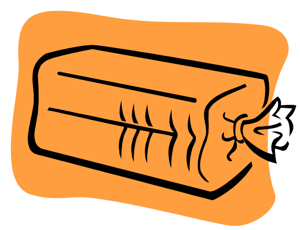
Brown's Bread: 0% extra free

Cost = 98p – (10% of 98p)

= 98p – 9.8p

= 88.2p

Grams = 790g



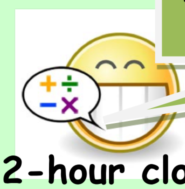
Cost per gram = 88p ÷ 790g

= <u>0.11p per gram</u>

The Swarbrick's special offer is the best value for money if you would normally use 3 loaves of bread before the bread went stale.

Shape, Space and Measures Time

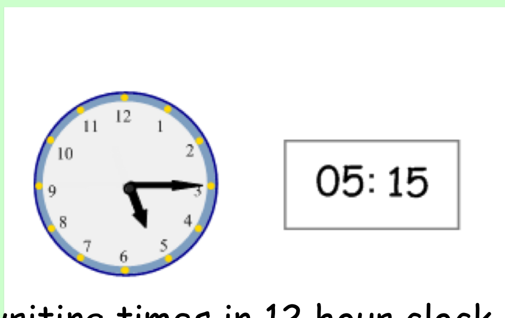
TIME



Time may be expressed in 12 or 24 hour

12-hour clock

Time can be displayed on a clock face, or digital clock.



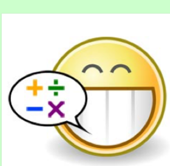
These clocks both show fifteen minutes past five, or quarter past f...

When writing times in 12 hour clock, we need to add a.m. or p.m. after the time.

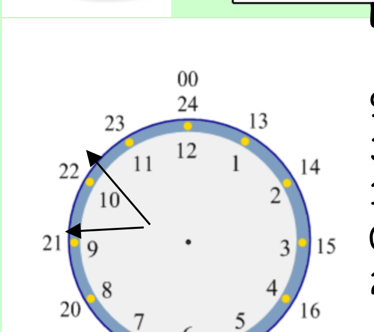
a.m. is used for times between midnight and 12 noon (morning)

p.m. is used for times between 12 noon and midnight (afternoon / evening).

24-hour clock



In 24 hour clock, the hours are written as numbers between 00 and 24. Midnight is expressed as 00 00, or 24 00. After 12 noon, the hours are numbered 13, 14, 15 … etc.



Examples

9.55 am → 09 55 hours

3.35 pm → 15 35 hours

12.20 am → 00 20 hours

02 16 hours → 2.16 am

20 45 hours → 8.45 pm

Time Periods

It is essential to know the number of months, weeks and days in a year, and the number of days in each month.

Time Facts

In 1 year, there are: 365 days (366 in a leap year)
52 weeks
12 months

The number of days in each month can be remembered using the rhyme:

"30 days hath September,
April, June and November,
All the rest have 31,
Except February alone,
Which has 28 days clear,
And 29 in each leap year."

Interpreting Timetables

Destination	Time	Time	Time	Time	Time	Time	Time	Time	Time
Thurso Business Park	0645	0745	0905	1005	1105	1205	1305	1405	1505
Olrig Street Job Centre	0650	0750	0910	1010	1110	1210	1310	1410	1510
Halkirk Sinclair Street	0705	0805	0925	1025	1125	1225	1325	1425	1525
Watten Post Office	0718	0818	0938	1038	1138	1238	1338	1438	1538
Haster Fountain Cottages	0725	0825	0945	1045	1145	1245	1345	1445	1545
Wick Somerfield bus terminal	0730	0830	0950	1050	1150	1250	1350	1450	1550
Wick Business park	0735	0835	0955	1055	1155	1255	1355	1455	1555
Wick Tesco Store	0736	0836	0956	1056	1156	1256	1356	1456	1556
Wick Airport Terminal	0741	0841	1001	1101	1201	1301	1301	1401	1601

Examples of Questions:

a) I want to be at Wick Airport by 2.30pm. What time must I catch the bus at Olrig Street Job Centre?

2.30pm is shown as 1430 h on the timetable
The most suitable bus arrives at Wick Airport at 1401
This leaves Olrig Street Job Centre at **1310 h**

b) The 0745 bus from Thurso Business Park is running 6 minutes late. What time does it reach Wick Tesco Store?

Add 6 minutes to the arrival time at Wick Tesco Store

This is 0836 h. <u>It arrives at **0842 h**</u>

How long does the first bus journey from Halkirk to Wick Business Park take?

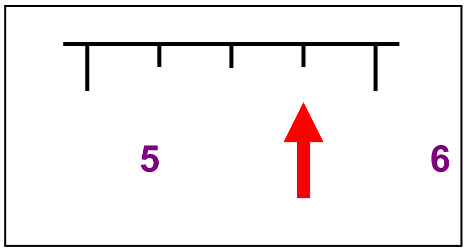
The bus leaves Halkirk at 0705 h and arrives at Wick Business Park at 0735 h.

The journey time is <u>**30 minutes.**</u>

Measurement

Reading scales

Scale 1



In this scale the difference between 5 and 6 is 1, and the space has been divided into 4, so each division represents 1 ÷ 4 = 0.25.

The arrow is pointing to 5 + 0.25 + 0.25 + 0.25 = 5.75

Scale 2 - a speedometer

The difference between 50 and 60 is 10 and the space has been divided into 2, so each division represents 10 ÷ 2 = 5.
The arrow is pointing to 50 + 5 = 55

Converting between units

The table shows some of the most common equivalences between different units of measure. Make sure you know these **conversions**.

Length	Weight	Capacity
	1 tonne = 1000kg	
1 km = 1000m	1kg = 1000g	
1m = 100cm = 1000mm	1g = 1000mg	1l = 100cl = 1000ml
1cm = 10mm		1cl = 10ml

If converting from a larger unit (eg m) to a smaller unit (eg cm), check what number of smaller units are needed to make 1 larger unit, then multiply that number with the relevant number of the larger units.

If converting from a smaller unit (eg cm) to a larger unit (eg m), check what number of smaller units are needed to make 1 larger unit, then divide that number into the relevant number of the larger units.

Remember: To convert from a larger unit to a smaller one, **multiply**.
To convert from a smaller unit to a larger one, **divide**.

Worked example
We know that 1m = 100cm

So, to convert from m to cm we multiply by 100, and to convert from cm to m we divide by 100.

Eg: 3.2m = **320cm** (3.2 x 100 = 320)
400cm = **4m** (400 ÷ 100 = 4)

Metric and imperial units

Imperial measures are old-fashioned units of measure. These days we have mostly replaced them with **metric units**, but despite our efforts to 'turn metric', we **still use many imperial units in our everyday lives.** It is therefore important that we are able to calculate **rough equivalents** between **metric and imperial units**.

Here are some conversions that you will need to know:

1 inch is about 2.5cm

1 foot is about 30cm

1kg is about 2.2 pounds

8km is about 5 miles

(1km is about 5/8 mile, and 1 mile is about 8/5km)

Worked example

We know that 1 mile is about 1.6 km.

To convert from miles to km, we multiply by 1.6.

To convert from km to miles, we divide by 1.6.

E.g. 20 litres = 32 km (20 x 1.6 = 32)

80 km/hr = 50 mph (80 ÷ 1.6 = 50)

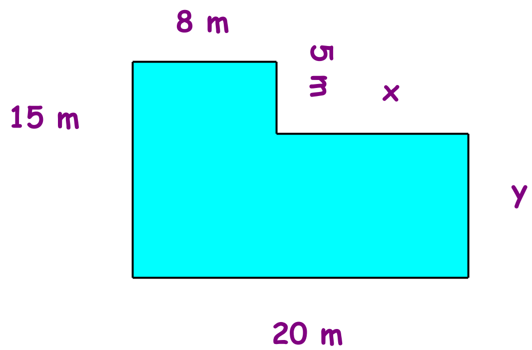
Perimeter (always measured in cm,mm,m,km,ft,in)

The perimeter of a shape is the length of its boundary or outside edges.

Think of a football pitch, If I walk around the edge of the pitch, the distance I walk is the perimeter of the field.

Example question

A plan of a play area is shown below:



a) Calculate the length of x and y

The length of the play area is 20m, so x = 20 - 8 =12m.The width of the play area is 15m, so y = 15 - 5 = 10m.

b) Calculate the perimeter of the play area.

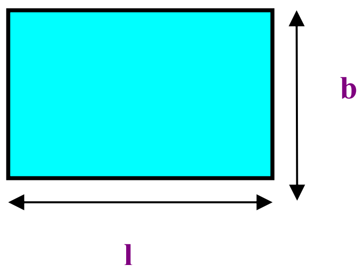
Perimeter = 20 + 15 +8 + 5 + 12 + 10

= 70 m

Area (always measured in cm^2, mm^2, m^2, km^2, ft^2, in^2)

Area of a rectangle

Area = l x w

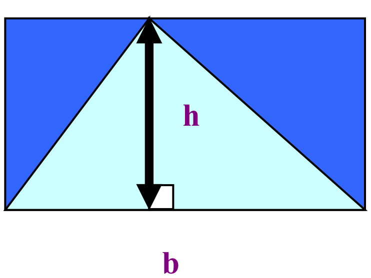


The area of a rectangle is its length multiplied by its width.

The formula is: **area = length x width**

Area of a triangle

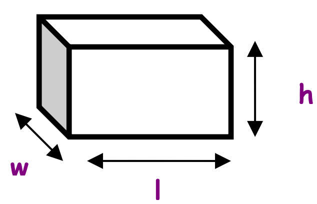
Look at the triangle below:



If you multiplied the base by the **perpendicular(at 90˚ to)** height, you would obtain the area of a rectangle. The area of the triangle is **half the area of the rectangle.** So to find the area of a triangle, we multiply the **base by the perpendicular height and divide by two.** The formula is:

Area =base x height divided by 2

Volume (always measured in cm^2, mm^2, m^2, km^2, ft^2, in^2

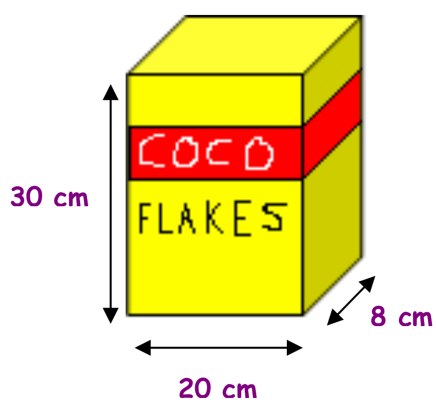


To find the volume of a cuboid we find the front (cross-sectional) area and multiply by its depth or width.

For example:

The volume of this cereal packet is $20 \times 30 = 600$ cm^2

Then $600 \times 8 =$ **$4800cm^3$**



Volume can also be measured in Litres. $1000cm^3$ = 1 Litre

Statistics

10 Commandments for drawing or plotting a graph

1. I shall always put a title on my graph.
2. I shall always think about which type of graph is best to use.
3. I shall always use a pencil and ruler to draw my axes.
4. I shall always try to fill my graph paper with my graph by choosing a suitable scale.
5. I shall always put the dependent variable (one that we measure or observe) on the y axis.
6. I shall always put the independent variable (the one we change) on the x axis.
7. I shall always label both axes
8. I shall always put the units on my axes
9. I shall always plot my points accurately using crosses.
10. I shall always draw a smooth curve or a straight line (with a ruler) where appropriate.

Data Tables

It is sometimes useful to display information in graphs, charts or tables.

Example 1 The table below shows the average maximum temperatures (in degrees Celsius) in Barcelona and Edinburgh.

	J	F	M	A	M	J	J	A	S	O	N	D
Barcelona	13	14	15	17	20	**24**	27	27	25	21	16	14
Edinburgh	6	6	8	11	14	17	18	18	16	13	8	6

The average temperature in June in Barcelona is 24°C

Frequency Tables are used to present information. We group large amounts of data into group or intervals.

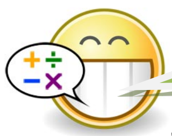
Example 2 Homework marks for Class 4B

27 30 23 24 22 35 24 33 38 43 18 29 28 28 27
33 36 30 43 50 30 25 26 37 35 20 22 24 31 48

<table>
<tr><th>Mark</th><th>Tally</th><th>Frequency</th></tr>
<tr><td>16 - 20</td><td>||</td><td>2</td></tr>
<tr><td>21 - 25</td><td>|||| ||</td><td>7</td></tr>
<tr><td>26 - 30</td><td>|||| ||||</td><td>9</td></tr>
<tr><td>31 - 35</td><td>||||</td><td>5</td></tr>
<tr><td>36 - 40</td><td>|||</td><td>3</td></tr>
<tr><td>41 - 45</td><td>||</td><td>2</td></tr>
<tr><td>46 - 50</td><td>||</td><td>2</td></tr>
</table>

Each mark is recorded in the table by a tally mark.
Tally marks are grouped in 5's to make them easier to read and count.

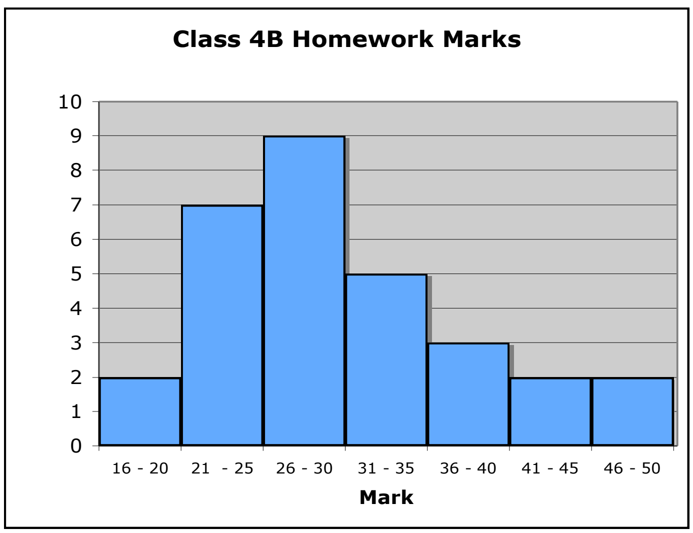
Bar Graphs



Bar graphs are often used to display data. The horizontal axis should show the categories or class intervals, and the vertical axis the frequency.

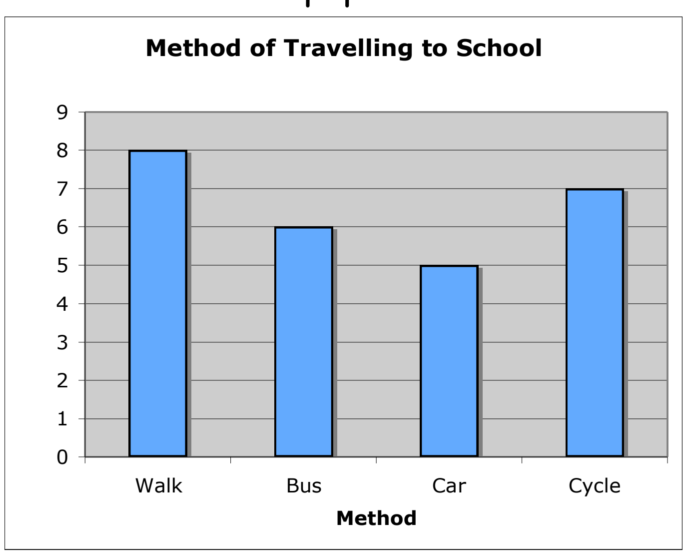
Example 1 The **frequency diagram** graph below shows the homework marks for Class 4B.

Number of Pupils



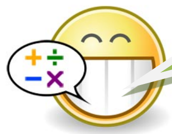
Example 2 A Bar chart to show how pupils travel to school

Number of Pupils



NOTICE this bar chart has gaps between as they are categories not groups.Continuous data(can take any value) is put into a frequency diagram, which has NO gaps.

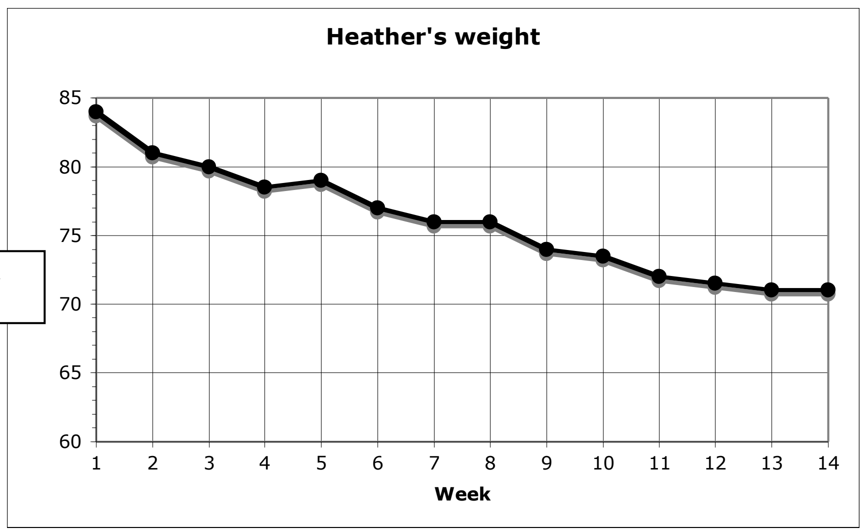
Line Graphs



Line graphs consist of a series of points which are plotted, then joined by a line.
The trend of a graph is a general

Example 1 The graph below shows Heather's weight over 14 weeks as she follows an exercise programme.

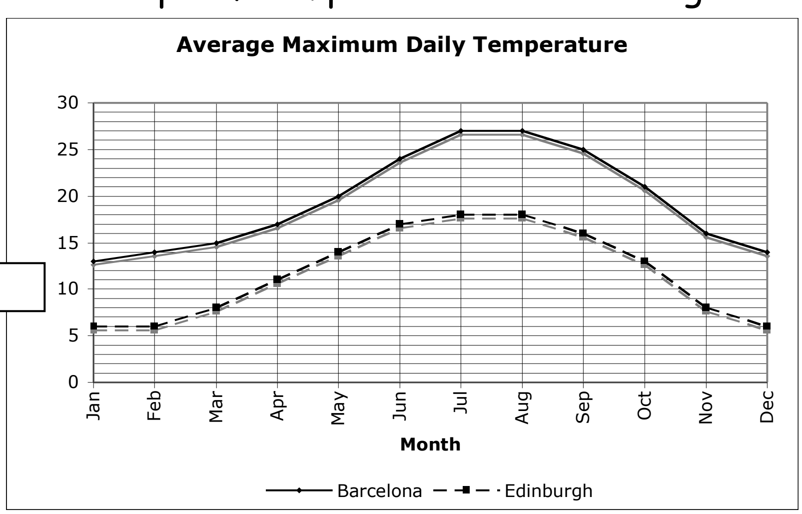
kg

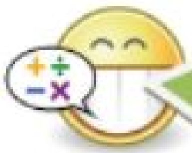


The trend of the graph is that her weight is decreasing.

Example 2 Graph of temperatures in Edinburgh and Barcelona.

^{0}C

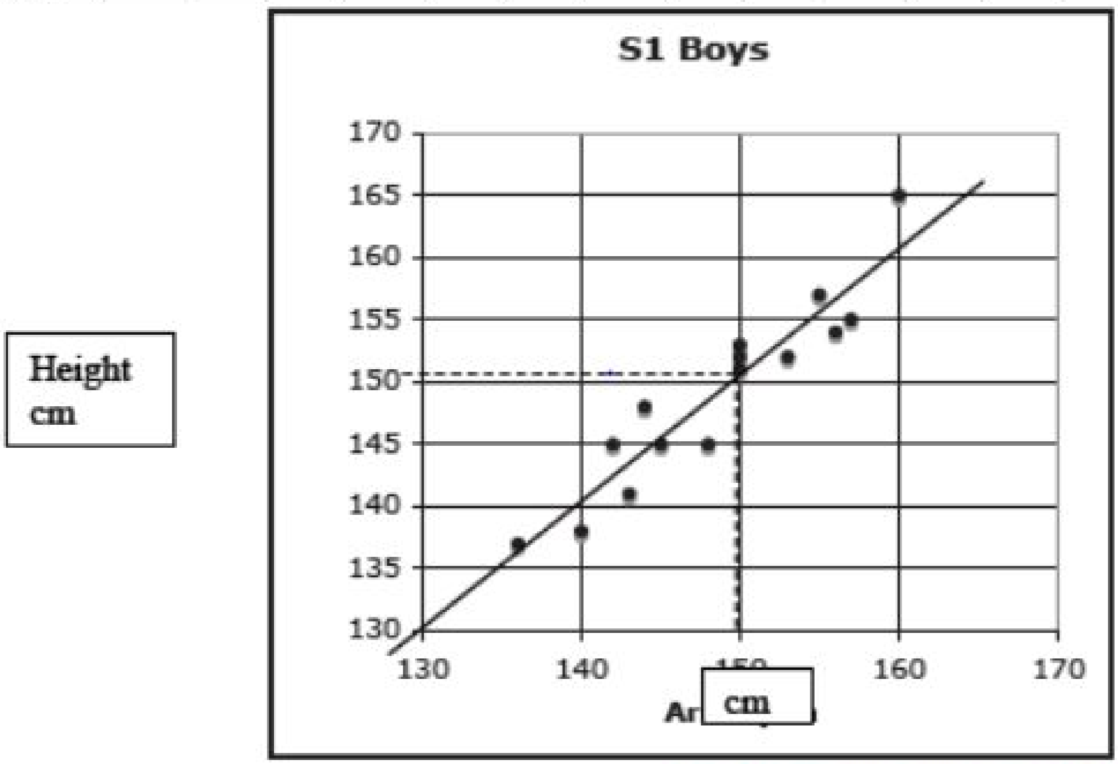




A scatter diagram is used to display the relationship between two variables.
A pattern may appear on the graph. This is called a **correlation**.

Example The table below shows the height and arm span of a group of first year boys. This is then plotted as a series of points on the graph below.

Arm Span (cm)	150	157	155	142	153	143	140	145	144	150	148	160	150	156	136
Height (cm)	153	155	157	145	152	141	138	145	148	151	145	165	152	154	137

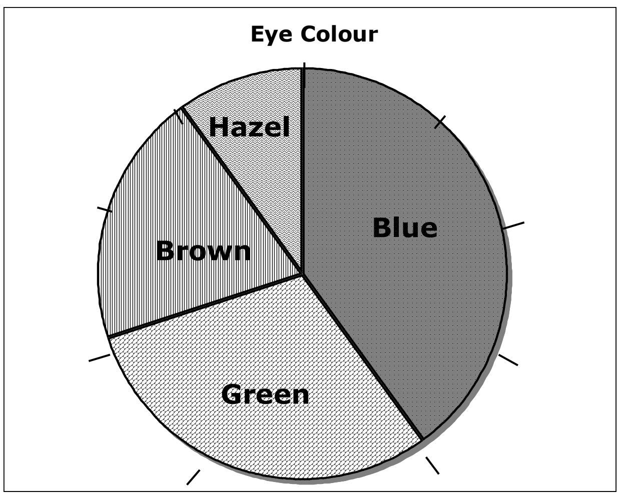


The graph shows a general trend, that **as the arm span increases, so does the height. This graph shows a <u>positive correlation.</u>**
The line drawn is called the **<u>line of best fit</u>**. This line can be used to provide estimates. For example, a boy of arm span 150cm would be expected to have a height of around 151cm.
Note that **<u>in some subjects</u>**, axes may need to start from zero.

Pie Charts

A pie chart can be used to display information. Each sector (slice) of the chart represents a different category. The size of each category can be worked out as a fraction of the total using the number of divisions or by measuring angles.

Example 30 pupils were asked the colour of their eyes. The results are shown in the pie chart below.



How many pupils had brown eyes?

The pie chart is divided up into ten parts, so pupils with brown eyes represent $\frac{2}{10}$ of the total.

$\frac{2}{10}$ of 30 = 6 so 6 pupils had brown eyes.

If no divisions are marked, we can work out the fraction by measuring the angle of each sector.

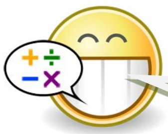
The angle in the brown sector is 72°.

so the number of pupils with brown eyes

$= \frac{72}{360} \times 30 = 6$ pupils.

If finding all of the values, you can check your answers - the pupils.

Drawing Pie Charts



On a pie chart, the size of the angle for each sector is calculated as a fraction of 360°.

Example: In an essay, the number of marks gained on an assignment IS 80 . This is split into Q1, Q2 etc. Draw a pie chart to illustrate the information.

Section of Paper	**Number of people**
Section 1	28
Section 2	24
Section 3	10
Section 4	12
Spelling Punctuation and grammar	6

Total Marks = 80

$$= \frac{28}{80} \rightarrow \frac{28}{80} \times 360^\circ = 126^\circ$$

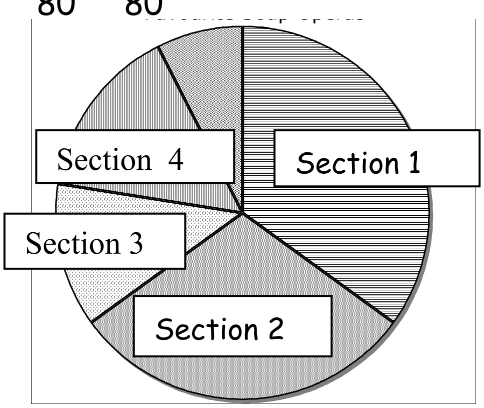
$$= \frac{24}{80} \rightarrow \frac{24}{80} \times 360^\circ = 108^\circ$$

$$= \frac{10}{80} \rightarrow \frac{10}{80} \times 360^\circ = 45^\circ$$

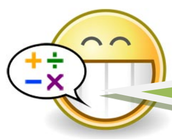
$$= \frac{12}{80} \rightarrow \frac{12}{80} \times 360^\circ = 54^\circ$$

$$= \frac{6}{80} \rightarrow \frac{6}{80} \times 360^\circ = 27^\circ$$

Check that the total = 360°



Averages



To provide information about a set of data, the average value may be given. There are 3 different types of **average** value – the mean, the median and the mode.

You can remember it by the following rhyme:

"HEY DIDDLE DIDDLE, THE MEDIAN'S IN THE MIDDLE, YOU ADD AND DIVIDE FOR THE MEAN. THE MODE IS THE ONE YOU SEE THE MOST, THE RANGE IS THE DIFFERENCE BETWEEN."

Mean is found by adding all the data together and dividing by the number of values.

Median is the middle value when all the data is written in numerical order (if there are two middle values, the median is half-way between these values).

Mode is the value that occurs most often.

Range is the range of a set of data is a measure of spread. = Highest value – Lowest value

Example The temperature each day, over 2 weeks is recorded in °C. Find the mean, median, mode and range of the results.

7, 9, 7, 5, 6, 7, 10, 9, 8, 4, 8, 5, 7, 10

Mean = $\frac{7+9+7+5+6+7+10+9+8+4+8+5+7+10}{14}$

= $\frac{102}{14} = 7.285...$ Mean = 7.3°C to 1 decimal place

Ordered values: 4, 5, 5, 6, 7, 7, 7, 7, 8, 8, 9, 9, 10, 10

Median = 7 °C

7 is the most frequent temperature, so **Mode = 7 °C**

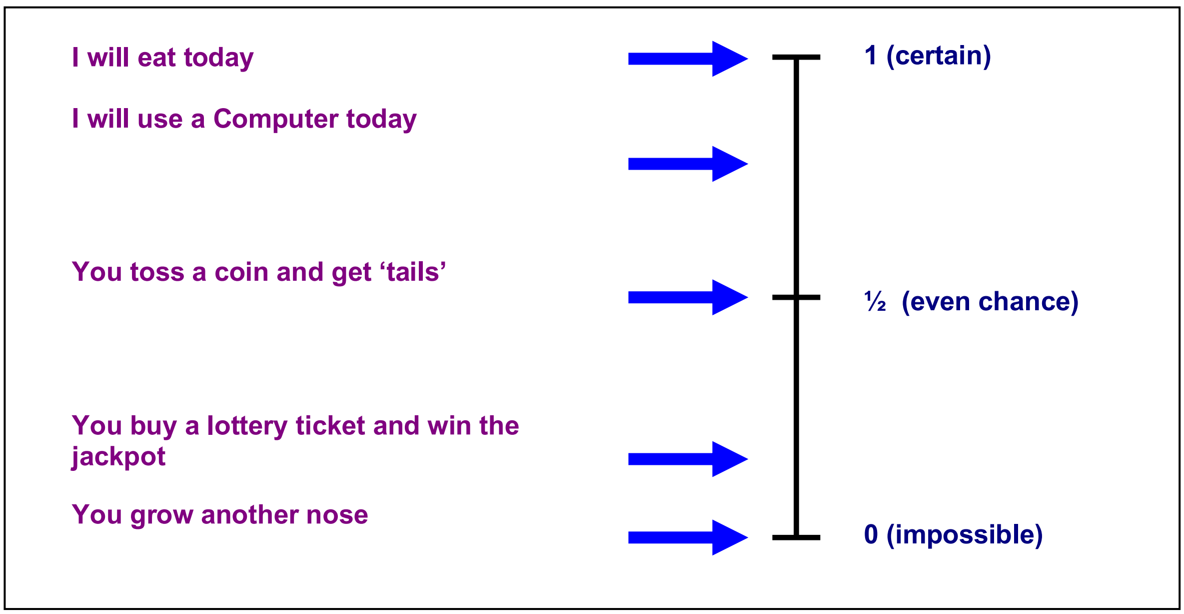
Range = 10 – 4 = 6

Probabilities

We often make judgments as to whether an event will take place, and use words to describe how probable that event is.
For example, we might say that it is likely to be sunny tomorrow, or that it is impossible to find somebody who is more than 3m tall, or it is unlikely I will win the lottery.

The probability scale

In maths we use numbers to describe probabilities. Probabilities can be written as fractions, decimals or percentages. We can also use a probability scale, starting at 0 (impossible) and ending at 1 (certain).



**When we throw a die (plural: dice), there are six possible different outcomes. It can show either 1, 2, 3, 4, 5 or 6.
But how many possible ways are there of obtaining an even number? Clearly, here are three: 2, 4 and 6.
We say that the probability of obtaining an even number is 3/6 (= 1/2 or 0.5 or 50%)**

The probability of an outcome = $\dfrac{\text{number of ways the outcome can happen}}{\text{total number of possible outcomes}}$

Example 1

How many outcomes are there for the following experiments? List all the possible outcomes.

a) Tossing a coin.

There are two possible outcomes (head and tail).

b) Choosing a sweet from a bag containing 1 red, 1 blue, 1 white and 1 black sweet.

There are four possible outcomes (red, blue, white and black).

c) Choosing a day of the week at random.

There are seven possible outcomes (Sunday, Monday, Tuesday, Wednesday, Thursday, Friday and Saturday).

Glossary of Terms

a.m.	(ante meridiem) Any time in the morning (between midnight and 12 noon). am = After midnight
Add; Addition (+)	To combine 2 or more numbers to get one number (called the sum or the total) Example: 12+76 = 88
Approximate	An estimated answer, often obtained by rounding to nearest 10, 100 or decimal place.
Area	Amount of surface
Average	Mean, Median and Mode
Bar Graph	One of the ways of presenting data in the form of a graph or chart.
Bargain	An item that has been bought at a reduced price which the customer believes to be a good deal.
Calculate	Find the answer to a problem. It doesn't mean that you must use a calculator!
Cuboid	Rectangular prism - see triangular prism
Cylinder	Circular prism – see triangular prism
Data	A collection of information (may include facts, numbers or measurements).
Deals	Another term for a special offer.
Decimal places	Places to the right of the decimal point. The first number to the right is the first decimal place.
Denominator	The bottom number in a fraction (the number of parts into which the whole is split).
Difference (-)	The amount between two numbers (subtraction).

	Example: The difference between 50 and 36 is 14 50 – 36 = 14
Discount	The amount of money that the price of an item has been reduced by , the amount taken off the original price.
Division (÷)	Sharing a number into equal parts. 24 ÷ 6 = 4
Double	Multiply by 2.
Equals (=)	Makes or has the same amount as.
Equivalent fractions	Fractions which have the same value. Example $\frac{6}{12}$ and $\frac{1}{2}$ are equivalent fractions
Estimate	To make an approximate or rough answer, often by rounding.
Evaluate	To work out the answer.
Even	A number that is divisible by 2. Even numbers end with 0, 2, 4, 6 or 8.
Factor	A number which divides exactly into another number, leaving no remainder. Example: The factors of 15 are 1, 3, 5, 15.
Frequency	How often something happens. In a set of data, the number of times a number or category occurs.
Greater than (>)	Is bigger or more than. Example: 10 is greater than 6. 10 > 6
Least	The lowest number in a group (minimum).
Less than (<)	Is smaller or lower than. Example: 15 is less than 21. 15 < 21.
Line Graph	One of the ways of presenting data in the form of a graph or chart.

Maximum	The largest or highest number in a group.
Mean	The arithmetic average of a set of numbers (see p32)
Median	Another type of average - the middle number of an ordered set of data (see p32)
Minimum	The smallest or lowest number in a group.
Minus (-)	To subtract.
Mode	Another type of average – the most frequent number or category (see p32)
Most	The largest or highest number in a group (maximum).
Multiple	A number which can be divided by a particular number, leaving no remainder. Example Some of the multiples of 4 are 8, 16, 48, 72
Multiply (x)	To combine an amount a particular number of times. Example 6 x 4 = 24
Negative Number	A number less than zero. Shown by a minus sign.
Numerator	The top number in a fraction.
Odd Number	A number which is not divisible by 2. Odd numbers end in 1 ,3 ,5 ,7 or 9.
Operations	The four basic operations are addition, subtraction, multiplication and division.
Order of operations	The order in which operations should be done. BIDMAS (see p9)
Outcome	An event that can happen
p.m.	(post meridiem) Any time in the afternoon or evening (between 12 noon and midnight). pm =

	past midday
Percentage off	The percentage of the original price.
Percentage of	The percentage of the original price that has been taken off.
Perimeter	Distance around the outside edge
Pie Chart	One of the ways of presenting data in the form of a graph or chart.
Possible	All the possible events that can happen
Place value	The value of a digit dependent on its place in the number. Example: in the number 1573.4, the 5 has a place value of 100.
Prime Number	A number that has exactly 2 factors (can only be divided by itself and 1). Note that 1 is not a prime number as it only has 1 factor.
Prism	3-dimensional shape with the same cross section along its length
Probability	How likely something is
Product	The answer when two numbers are multiplied together. Example: The product of 5 and 4 is 20.
Regular Price	The original price that an item has been advertised for before a special offer or discount has been.
Remainder	The amount left over when dividing a number.
Sale Price	The new price an item costs after a discount or special offer.
Scatter	One of the ways of presenting data in the form

Graph	of a graph or chart.
Share	To divide into equal groups.
Significant Figure	The first non-zero figures in a number which give the most information about the size of the number.
Sphere	A 3D Solid circular shape
Stem & Leaf Diagram	Different ways of presenting data in the form of a graph or chart.
Sum	The total of a group of numbers (found by adding).
Table	Different ways of presenting data in the form of a graph or chart.
Timetable	A table showing the times that someone or something is planned to arrive and depart.
Total	The sum of a group of numbers (found by adding).
Triangular Prism	3-dimensional shape with a triangular cross section along its length
Volume	Amount of space inside a shape or the amount of space an object takes up